Pure math – Model 1

1. If $(1, \omega, \omega^2)$ are the cubic roots of 1, then $(\omega + \omega^2 + \cdots + \omega^{100}) = \cdots$

a) 1 b) ω c) ω^2 d) zero

2. If θ, θ, β are directed angles of $\overrightarrow{A}$ and $\sin^2 \beta = 3\sin^2 \theta$, then $\cos^2 \theta = \cdots$

a) $\frac{3}{5}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{2}$

3. If $n = \ln x, y = e^n$, then $\frac{dy}{dx} = \cdots$

a) Zero b) 1 c) 2 d) 3

4. $\int 6xe^{3x^2+1} \cdot dx = \cdots + c$

a) e^{x^2+1} b) e^{3x^2} c) e^{3x^2+1} d) $\frac{1}{e^{x^2+1}}$

5. In the expansion of $(3 + 2x)^8 + (3 - 2x)^8$ at $x = \frac{1}{6}$, Then middle term $= \cdots$

a) 110 b) 120 c) 130 d) 140

6. If the point $(k, 4,5)$ is at equal distances from the x and z axes, then $k = \cdots$

a) ± 1 b) ± 3 c) ± 4 d) ± 5

7. If $x^2 y^3 = 8$, then $\frac{dy}{dx} = \cdots$ at $x = -1$

a) $\frac{4}{3}$ b) $\frac{-4}{3}$ C) $\frac{3}{4}$ d) $\frac{1}{2}$

8. $\int \frac{(\ln x)^2}{x} dx = \cdots + c$

a) $\frac{1}{3}(\ln x)^3$ b) $\frac{1}{2}(\ln x)^3$ c) $\ln x$ d) $\ln x^2$

9. If the middle term in the expansion of $(1+x)^{10}$ is twice the seventh term, then $x = \cdots$

a) 0.2 b) 0.4 c) 0.6 d) 0.8

10. If $\overrightarrow{AB} = -3\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$, and $\overrightarrow{BC} = \hat{\jmath} + 5\hat{k}$, then $\| \overrightarrow{AC} \| = \cdots$

a) 8 b) 10 c) 12 d) 13

11. If $x = 3t^2 - 1, y = t^3 + 2$, then $\frac{d^2y}{dx^2} = \cdots$ at $t = 4$

a) 48 b) $\frac{1}{24}$ c) $\frac{1}{48}$ d) 24

12. The volume of the solid generated by rotating the region bounded by the curve $y = x(x-2)$ a complete cycle about the x-axis $= \cdots$ cubic unit

a) $\frac{16}{15}\pi$ b) $\frac{19}{15}\pi$ c) $\frac{17}{15}\pi$ d) $\frac{15}{17}\pi$

13. The trigonometric form of the complex number $z = \frac{5-\sqrt{3}i}{\sqrt{3}-2i}$ is ....

a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ b) $\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

c) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ d) $3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

14. The equation of the plane passing through the point (1,2,3) and parallel to both the x and y axes is ...

a) $x + y = 3$ b) $x = 1$ c) $y = 2$ d) $z = 3$

15. A point is moving according to the relation $S = 3t^3 + 3t^2 - 4$, then $\frac{ds}{dt} = \cdots$ at $t = 3$

a) 77 b) 88 c) 99 d) 111

16. The two square roots of the number $z = 3 + 4i$ is ...

a) $\pm(2 + i)$ b) $\pm(2 + \sqrt{3}i)$

c) $\pm(1 + \sqrt{3}i)$ d) $\pm(1 + i)$

17. The direction vector of the straight line $\frac{x-2}{3} = \frac{y+3}{2}, z = 4$ is ...

a) $(3,2,4)$ b) $(3,2,0)$ c) $(2,-3,4)$ d) $(2,-3,0)$

18 If $f: f(x) = \sqrt[3]{x^2 - 6x}$, then the number of critical points of the curve of f is ...

a) Zero b) 1 c) 2 d) 3

Essay Questions:

19. If $k \in R$, then find the value of

$$\left(k - \frac{k+1}{\omega+1} + \omega^2(k + 1)\right)^8$$

20. The perimeter of a circular sector is 30 cm, find its radius when its area is maximum.

Model (1) 2024/2025

Pure Mathematics

1 $\omega+\omega^2+\omega^3+\cdots\cdots+\omega^{100}$

form a Geometric Series

$a = 1^{st}$ term $= \omega$

$r =$ Common ratio $= \omega$

$l =$ last term $= \omega$

and $S_n = \dfrac{a-lr}{1-r}$ Sum. of Geom. Series

$\therefore S_n = \dfrac{\omega - \omega^{100}\times\omega}{1-\omega} = \omega\dfrac{1-\omega^{100}}{1-\omega}$

$= \omega\dfrac{1-\omega}{1-\omega} = \omega.$

where $\omega^{100} = \omega.$ (b)

2 $\theta_x = \theta_y = \theta$, $\theta_z = \beta$

$\because \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\therefore \cos^2\theta + \cos^2\theta + \cos^2\beta = 1$

$2\cos^2\theta + \cos^2\beta = 1$

$2\cos^2\theta = 1 - \cos^2\beta$

$2[1-\sin^2\theta] = \sin^2\beta$

$2 - 2\sin^2\theta = 3\sin^2\theta$

$2 = 5\sin^2\theta$

$\sin^2\theta = \dfrac{2}{5}$

$1-\cos^2\theta = \dfrac{2}{5}$

$\therefore \cos^2\theta = 1 - \dfrac{2}{5} = \dfrac{3}{5}.$ (a)

3 If $n = \ln x$, $y = e^n$

$\therefore y = e^{\ln x} = x$

$\therefore \dfrac{dy}{dx} = 1.$ (b)

4 $\int 6x\, e^{3x^2+1}\, dx$

$\because f(x) = 3x^2+1$, then $f'(x) = 6x$

$= e^{3x^2+1} + c.$ (c)

5 $(3+2x)^8 + (3-2x)^8$

$= 2[T_1 + T_3 + \underline{T_5} + T_7 + T_9]$

$\therefore$ the middle term $= 2T_5$

$= 2\times {}^8C_4 (2x)^4 (3)^{8-4}$

Put $x = \dfrac{1}{6}$

$\therefore 2T_5 = 2\, {}^8C_4 \left(2\times\dfrac{1}{6}\right)^4 (3)^4$

$= 140.$ (d)

6 $\because$ distance from x-axis $=$ distance from z axis

$\therefore \sqrt{4^2+5^2} = \sqrt{k^2+4^2}$ "Squaring"

$16+25 = k^2+16$

$k^2 = 25$

$\therefore k = \pm 5.$ (d)

7 $x^2 y^3 = 8$

diff. bothe sides w.r.t x.

$$2x y^3 + 3x^2 y^2 \frac{dy}{dx} = 0$$

at $x = -1$, then $y = 2$

$$\therefore 2(-1)(2)^3 + 3(-1)^2(2)^2 \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{4}{3} \cdot \text{ (a)}$$

8 $\int \frac{(\ln x)^2}{x} dx = \int \underbrace{\frac{1}{x}}_{f'(x)} \underbrace{(\ln x)}_{f(x)}^2 dx$

$$= \frac{1}{3} (\ln x)^3 + C \cdot \text{ (a)}$$

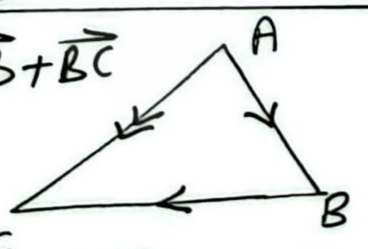
9 order of the middle term $= \frac{10}{2} + 1 = 6$

$$\therefore T_6 = 2\, T_7 \Rightarrow \frac{T_7}{T_6} = \frac{1}{2}$$

$$\frac{10-6+1}{6} \times \frac{x}{1} = \frac{1}{2}$$

$$\therefore x = \frac{3}{5} = 0.6 \cdot \text{ (c)}$$

10 $\because \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$



$\therefore \overrightarrow{AC} = (-3, 4, 12)$

then $\|\overrightarrow{AC}\| = \sqrt{9 + 16 + 144} = 13 \cdot$ (d)

11 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3t^2}{6t}$

$$\therefore \frac{dy}{dx} = \frac{t}{2} \text{ or } \frac{1}{2}t$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{2} \times \frac{dt}{dx} = \frac{1}{2} \times \frac{1}{6t}$$

$$= \left[\frac{1}{12t}\right]_{\text{at } t=4} = \frac{1}{48} \cdot \text{ (c)}$$

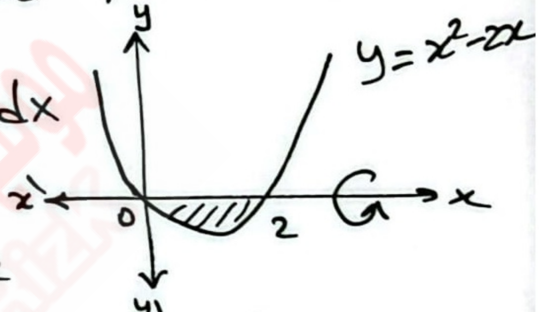
12 $y = x(x-2) = x^2 - 2x$

first: find the Points of intersection with x-axis

then put $y = 0$

then $x = 0$, $x = 2$

$$V = \pi \int_0^2 y^2 \, dx$$



$$V = \pi \int_0^2 (x^2 - 2x)^2 \, dx$$

$$= \frac{16}{15} \pi \text{ cubic unit. (a)}$$

13 $Z = \frac{5 - \sqrt{3} i}{\sqrt{3} - 2i} \times \frac{\sqrt{3} + 2i}{\sqrt{3} + 2i}$

$$= \frac{5\sqrt{3} + 10i - 3i - 2\sqrt{3} i^2}{3 + 4}$$

$$= \frac{5\sqrt{3} + 7i + 2\sqrt{3}}{7} = \frac{7\sqrt{3} + 7i}{7}$$

$$= \sqrt{3} + i \longrightarrow (+, +) \text{ 1st quad.}$$

$$|z| = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

and $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$ or $\frac{\pi}{6}$

$$\therefore Z = 2\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right) \cdot \text{ (c)}$$

14 ∵ Plane // xy-plane

∴ The equation of the plane is

$$z = z_1$$

where $x_1 = 1$, $y_1 = 2$, $z_1 = 3$

∴ The equation is $z = 3$. (d)

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15 $S = 3t^3 + 3t^2 - 4$

∴ $\frac{ds}{dt} = 9t^2 + 6t$

at $t = 3$; ∴ $\frac{ds}{dt} = 9(3)^2 + 6(3)$

$= 99$. (c)

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16 ∵ $\sqrt{x+yi} = \pm\left(\sqrt{\frac{r+x}{2}} \pm i\sqrt{\frac{r-x}{2}}\right)$

where $r = \sqrt{x^2+y^2}$

$\sqrt{z} = \sqrt{3+4i} = \pm\left(\sqrt{\frac{5+3}{2}} + i\sqrt{\frac{5-3}{2}}\right)$

where $r = \sqrt{3^2+4^2} = 5$

∴ $\sqrt{3+4i} = \pm(2+i)$. (a)

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17 $\vec{r} = (2,-3,4) + t(3,2,0)$

∴ $\vec{d} = (3,2,0)$. (b)

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18 $f(x) = \sqrt[3]{x^2 - 6x} \Rightarrow$ domain $= \mathbb{R}$

$f'(x) = \frac{2x-6}{3\sqrt[3]{(x^2-6x)^2}} \Rightarrow x \neq 0, 6$

Put $f'(x) = 0$: ∴ $2x - 6 = 0$

$x = 3$

∴ $f(x)$ have a critical points at

$x = 0$, $x = 6$, $x = 3$

then the number of critical points of the curve $f(x)$ is 3 points. (d)

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19 $\left(k - \frac{k+1}{\omega+1} + \omega^2(k+1)\right)^8$

$= \left(k - \frac{k+1}{-\omega^2} + \omega^2(k+1)\right)^8$

$= \left(k + \omega(k+1) + \omega^2(k+1)\right)^8$

$= \left(k + k\omega + \omega + k\omega^2 + \omega^2\right)^8$

$= (\omega + \omega^2)^8 = (-1)^8 = 1$.

where $k + k\omega + k\omega^2 = 0$

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20 ∵ Perimeter $= 2r + L = 30$

and Area $= \frac{1}{2} L r$

$A(r) = \frac{1}{2} r(30 - 2r)$

$= 15r - r^2$

$\frac{dA}{dr} = 15 - 2r \Rightarrow$ Put $\frac{dA}{dr} = 0$:

$\frac{d^2A}{dr^2} = -2$; ∴ $r = \frac{15}{2}$ cm

(-ve) ⇒ $A(x)$ has a max. value at $r = \frac{15}{2}$ cm

∴ $r = 7.5$ cm.